アブスト

A distributed network is modeled by a graph having n nodes (processors) and diameter D. We study the time complexity of approximating weighted (undirected) shortest paths on distributed networks with a O (log n) bandwidth restriction on edges (the standard synchronous CONGEST model). The question whether approximation algorithms help speed up the shortest paths and distance computation (more precisely distance computation) was raised since at least 2004 by Elkin (SIGACT News 2004). The unweighted case of this problem is well-understood while its weighted counterpart is fundamental problem in the area of distributed approximation algorithms and remains widely open. We present new algorithms for computing both single-source shortest paths (SSSP) and all-pairs shortest paths (APSP) in the weighted case.

Our main result is an algorithm for SSSP. Previous results are the classic O(n)-time Bellman-Ford algorithm and an Õ(n1/2+1/2k + D)-time (8k⌈log(k + 1)⌉ --1)-approximation algorithm, for any integer k ≥ 1, which follows from the result of Lenzen and Patt-Shamir (STOC 2013). (Note that Lenzen and Patt-Shamir in fact solve a harder problem, and we use Õ(·) to hide the O(poly log n) term.) We present an Õ (n1/2D1/4 + D)-time (1 + o(1))-approximation algorithm for SSSP. This algorithm is sublinear-time as long as D is sublinear, thus yielding a sublinear-time algorithm with almost optimal solution. When D is small, our running time matches the lower bound of Ω(n1/2 + D) by Das Sarma et al. (SICOMP 2012), which holds even when D=Θ(log n), up to a poly log n factor.

As a by-product of our technique, we obtain a simple Õ (n)-time (1+ o(1))-approximation algorithm for APSP, improving the previous Õ(n)-time O(1)-approximation algorithm following from the results of Lenzen and Patt-Shamir. We also prove a matching lower bound. Our techniques also yield an Õ(n1/2) time algorithm on fully-connected networks, which guarantees an exact solution for SSSP and a (2+ o(1))-approximate solution for APSP. All our algorithms rely on two new simple tools: light-weight algorithm for bounded-hop SSSP and shortest-path diameter reduction via shortcuts. These tools might be of an independent interest and useful in designing other distributed algorithms.